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any point is equal to that which would be produced by a wire of the same weight per unit length, hanging vertically and being of the same length as the ordinate of the point where the tension is measured, the equation of the curve being of the form,

$$y = \frac{a}{2}\left(e^{x/a} + e^{-x/a}\right).$$

Hence, the equation of the catenary curve assumed by the wire in the above problem will be

$$y = 700\left(e^{x/1400} + e^{-x/1400}\right),$$

the unit of distance being one yard.

The amount of dip or sag in a symmetrical segment of horizontal projection, $2h$, will be

$$\begin{aligned} d &= \frac{a}{2}\left(e^{h/a} + e^{-h/a}\right) - a \\ &= \frac{h^2}{2a} + \frac{h^4}{24a^3} + \frac{h^6}{720a^5} + \cdots. \end{aligned}$$

In our problem $d = 1$, $a = 1400$, and a very close first approximation is given by neglecting all terms in the series after the first. Hence, $h = 52.915$, approximately. Using more terms the result correct to three places is $h = 52.912$. The distance between the poles, correct to the nearest tenth of a foot is 317.5 feet.

Also solved by ROGER A. JOHNSON.

2671. Proposed by ARTEMAS MARTIN, Washington, D. C.

Find two rectangular parallelopipedons whose edges are rational whole numbers and whose solid diagonals are also rational whole numbers and equal.

SOLUTION BY S. A. COREY, Albia, Iowa.

We have the identities

$$\begin{aligned} (x^2 + y^2 + u^2 + v^2)^2 &= (x^2 - y^2 - u^2 + v^2)^2 + (2xy - 2uv)^2 + (2ux + 2vy)^2 \\ &= (x^2 - y^2 + u^2 - v^2)^2 + (2xy + 2uv)^2 + (2vx - 2uy)^2 \\ &= (x^2 + y^2 - u^2 - v^2)^2 + (2ux - 2vy)^2 + (2vx + 2uy)^2. \end{aligned}$$

By letting x, y, u, and v represent rational whole numbers any number of solutions of the problem may be obtained. One such solution is obtained by letting $x = 1$, $y = 2$, $u = 5$, and $v = 7$, and we find *three* parallelopipedons fulfilling the requirements of the problem, with edges 21, 66, 38; 27, 74, 6; and 69, 18, 34, respectively, the solid diagonal of each being 79. The three smallest have edges, 1, 2, 2; 2, 4, 4; and 2, 3, 6, respectively.

Also solved by L. E. LUNN, W. P. DURFEE, and H. L. OLSON.

2673. Proposed by WILLIAM O. BEAL, University of Minnesota.

A plane through the center of an oblate spheroid makes an angle, i, with the plane of its equator. Express the eccentricity, e', of this section in terms of the eccentricity, e, of a meridian section and the angle, i.

SOLUTION BY C. A. BARNHART, Colorado College.

Let the equation of the oblate spheroid be $x^2/b^2 + y^2/b^2 + z^2/a^2 = 1$, $(b > a)$, of which the z-axis is the axis of revolution. Let $OP = b'$ be the semi-minor axis of the given plane section of which b is the semi-major axis. Then P will be a point on the meridian section, which may be assumed to lie in the yz-plane and to have the parametric equations:

$$z = a \sin \theta, \qquad y = b \cos \theta;$$

and the eccentricity,

$$e = \frac{\sqrt{b^2 - a^2}}{b}.$$

Since

$$e' = \frac{\sqrt{b^2 - b'^2}}{b}$$

and

$$b' = \sqrt{y^2 + z^2} = \sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta},$$

$$e' = \frac{\sqrt{b^2 - b^2 \cos^2 \theta - a^2 \sin^2 \theta}}{b} = \frac{\sqrt{(b^2 - a^2)(1 - \cos^2 \theta)}}{b} = e \sin \theta.$$

But $z/y = \tan i = (a/b) \tan \theta$, or $\tan \theta = (b/a) \tan i$. Therefore,

$$\sin \theta = \frac{b \tan i}{\sqrt{a^2 + b^2 \tan^2 i}} = \frac{b \tan i}{\sqrt{b^2 - b^2e^2 + b^2 \tan^2 i}} = \frac{\tan i}{\sqrt{\sec^2 i - e^2}}.$$

Substituting for $\sin \theta$, we have

$$e' = \frac{e \tan i}{\sqrt{\sec^2 i - e^2}}.$$

Also solved by Roger A. Johnson, Elbert H. Clarke, H. L. Olson, and the Proposer.

2674. Proposed by J. O. MAHONEY, Dallas, Texas.

If two sides of a triangle differ by less than a certain length, e, the two opposite angles will differ by less than a certain quantity λ, expressed in degrees, such that $\lambda < 61e/a$ where a expresses, with a possible error e, the length of the apparently equal sides of the triangle.

Solution by Roger A. Johnson, Hamline University.

This theorem as stated, is not true. Consider, for instance, the triangle

$$a = 1001, \qquad b = 1000, \qquad c = 99.$$

Here,

$$A = 87° 44' 32'', \qquad B = 86° 35' 10'', \qquad C = 5° 40' 18''.$$

Now, considering the nearly equal angles A and B, we have, in fact, $\lambda = 1.157$, whereas by the formula given, we should have $\lambda = .061$ or less. This is an extreme case, but it will be found that in any triangle in which the nearly equal angles are greater than about 50°, the formula does not hold.

As a matter of fact, the correct expression is

$$\lambda = \frac{180}{\pi} \frac{e}{a} \tan A,$$

where A represents the larger of the nearly equal angles. We will not consider the case that either of these two angles exceeds or equals 90°.

If a and b are two sides of a triangle, α, β, the opposite angles, we have

$$\sin \beta = \frac{\sin \alpha}{a} \cdot b;$$

whence

$$\cos \beta d\beta = \frac{\sin \alpha}{a} db,$$

in radians, or

$$d\beta = \frac{180°}{\pi} \cdot \frac{\sin \alpha}{\cos \beta} \frac{db}{a}$$

For our problem, $d\beta = \lambda$, $db = e$, and $(\sin \alpha)/(\cos \beta)$ may be replaced by the greater of the values $\tan \alpha$, $\tan \beta$, yielding the inequality given above.

Also solved by Elijah Swift and Paul Capron.

2675. Proposed by E. B. ESCOTT, Kansas City, Mo.